Roll No. ....

Total No. of Questions : **9**] [Total No. of Printed Pages : **4**

(2034)

BCA (CBCS) RUSA IInd Semester Examination

3768

MATHEMATICS–II

Paper : BCA-0201

Time : 3 Hours] **[Maximum Marks : 70**

Note :– Attempt *one* question from each Unit. Q. No. **9** is compulsory.

Unit–I

1. (a) Verify Cauchy's Mean Value Theorem for the functions

$$f(x) = \log x \text{ and } g(x) = \frac{1}{x}$$

in the interval $[1, e]$.

(b) State and prove Lagrange's Mean Value Theorem. 5,5

2. (a) If $y = (\sin^{-1} x)^2$, find $y_n(0)$.

(b) If $p^2 = a^2 \cos^2 \theta + b^2 \sin^2 \theta$, prove that :

$$p + \frac{d^2 p}{d\theta^2} = \frac{a^2 b^2}{p^3}$$ 5,5

Unit–II

3. (a) Show that for any two integers a and b, with $a > 0$, there exist unique integers q and r such that $b = aq + r,\ 0 \le r < |a|$.

(b) If a and b are relatively prime integers, then any common divisor of ac and b is a divisor of c. 5,5

4. (a) Solve $2x + 7y \equiv 5 \pmod{12}$

(b) Find the g.c.d. of 275 and 200 and express it in the form $275x + 200y$. 5,5

Unit–III

5. (a) Let Q* denotes the set of all relational numbers expect 1, then show that Q* forms an infinite abelian group under the operation 'o' defined by $a \ o \ b = a + b - ab \ \forall \ a, b \in Q^*$.

(b) If in a group G, $a^5 = e$ and $aba^{-1} = b^2$ for all $a, b \in G$. Prove that if $b \neq e$, then $b^{31} = e$. 5,5

6. (a) Show that the multiplicative group $[1, -1, i, -i\}$ formed by the fourth roots of unity is a cyclic group.

(b) If G is a group under the binary operation * and a, b are elements of G, then prove that :

$$(a * b)^{-1} = b^{-1} * a^{-1} \ \forall \ a, b \in G.$$ 5,5

Unit–IV

7. (a) If R is a ring with identity such that :

$$(xy)^2 = x^2y^2 \ \forall \ x, y \in R$$

then show that R is a commutative ring.

(b) Let R be the set of all reals :

$$\text{Take } F = \left\{ \begin{bmatrix} a & b \\ -b & a \end{bmatrix} : a, b \in R \right\}$$

Then prove that (F, +, .) is a field. 5,5

8. (a) Prove that the set $G = \left\{ a + \sqrt{2}b \mid a, b \in Q \right\}$ where Q is the set of rationals, is a ring.

(b) Give an example of a division ring which is not a field. 5,5

Unit–V

(Compulsory Question)

9. (i) State Cauchy's mean value theorem.

(ii) If $y = e^{ax} \cos(bx + c)$ then y_n = ...

(iii) State Leibnitz's theorem.

(iv) $8^n - 3^n$ is not divisible by 5. (True/False)

(v) Find g.c.d. (121, 550, 770).

(vi) Find the remainder when 2^{340} is divided by 341.

(vii) Does the set of positive irrational numbers form a group under multiplication ? (Yes/No)

(viii) If $(ab)^2 = a^2b^2$, $\forall\ a, b \in G$, where G is a group, then show that G must be an abelian group.

(ix) Every cyclic group is abelian. (True/False)

(x) Identify which one from the following is not a field : 3×10=30

(N, +, .), (Q, +, .) and (C, +, .)